

# About possible influence of birefringence effect on processes of production (photoproduction, electroproduction) of vector mesons (particles with the spin $S \geq 1$ ) in nuclei

V.G. Baryshevsky

Research Institute for Nuclear Problems, Belarusian State University,  
11 Bobruyskaya Str., Minsk 220050, Belarus,  
e-mail: bar@inp.minsk.by

February 1, 2008

## Abstract

It is shown that birefringence effect influences on production of particles with the spin  $S \geq 1$  at collisions of high energy particles.

## 1 INTRODUCTION

Collision of high energy particles (proton, electron,  $\gamma$ -quanta, nucleus) with nucleus yields a lot of hadronic processes inside nuclei, which are accompanied by appearance of secondary particles (vector mesons,  $\Omega^-$  hyperons and so on). In particular the processes of photoproduction (electroproduction) of vector mesons by nuclei have been studying since the photoproduction vertex for hadronic probes inside nuclei is well known and the analysis of the results is simple and more reliable.

Moreover, experiments demonstrate production of both longitudinally (L) and transversally (T) polarized vector mesons and L/T-ratios depend on  $Q^2$  [1].

According to [2] consideration of photoproduction of vector mesons inside nuclei and their rescattering via strong interactions within Glauber multiple scattering theory allows to consider many peculiarities of the process important for understanding unconventional effects, like color transparency. In [2] (as well as in others) theoretical consideration of processes of rescattering of produced particles inside the nucleus is done without taking into account possible influence of produced particle spin on rescattering inside the nucleus. For the first glance there is no necessity to consider spin effects, caused by rescattering inside the nucleus, because usually nuclei are nonpolarized. However, this is a hasty conclusion.

In [3–5] it was shown that birefringence phenomenon appears when a deuteron (or a particle with spin  $S \geq 1$ ) passes through homogenies and isotropic matter. Birefringence phenomenon is the effect of spin rotation around momentum direction and spin dichroism (i.e. dependence of absorbtion coefficient on spin direction with respect to deuteron momentum). These effects are aroused due to intrinsic anisotropy, which is inherent for particles with spin  $S \geq 1$ . In particular this is well known for deuterons [6] and  $\Omega^-$ -hyperons [7], for which the ground state is the mix of  $S$  and  $D$  waves.

For a particle, produced inside a nucleus and moving though the nuclear matter, the conception of refraction index can be applied, too.

Birefringence effect can be described by the spin-dependent index of refraction:

$$\hat{n} = 1 + \frac{2\pi\rho}{k^2} \hat{f}(0), \quad (1)$$

where  $\rho$  is the density of scatterers in matter (the number of scatterers in 1 cm<sup>3</sup>),  $k$  is the particle wavenumber,  $\hat{f}(0)$  is used to denote the amplitude of zero-angle elastic coherent scattering of a particle by a scattering center, this amplitude is an operator acting in the particle spin space.

Refraction of particles in matter (either conventional or nuclear) implies existence of optical pseudopotential [3,4]:

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}\hat{f}(0), \quad (2)$$

where  $m$  particle mass and  $\gamma$  is its Lorentz factor.

For a particle with the spin 1 (deuteron, vector mesons) the amplitude of zero-angle elastic coherent scattering by a nonpolarized scatterer can be expressed in the following general form:

$$\hat{f}(0) = d + d_1(\vec{S}\vec{n})^2, \quad (3)$$

where  $\vec{S}$  is the operator of particle spin,  $\vec{n} = \frac{\vec{k}}{k}$  is the unit vector along particle momentum. The angle of spin rotation is determined by  $Re d_1$ , while  $Im d_1$  describes dichroism. Therefore, study of birefringence effect allows to find the spin-dependent part  $d_1$  of the amplitude of zero-angle elastic coherent scattering.

If scatterers are polarized, the expression for the scattering amplitude includes also terms depending on the scatterer polarization [3,4]. In particular, for  $\gamma$ -quanta passing through matter with polarized nuclei the amplitude of zero-angle scattering can be expressed as follows:

$$\hat{f}_{\gamma N}(0) = A(\vec{e}'^* \vec{e}) + iG[\vec{e}'^* \times \vec{e}] \vec{P} + Be_i'^* e_k Q_{ik} + Dn_{\gamma i} n_{\gamma k} Q_{ik}, \quad (4)$$

where  $\vec{e}$  is the vector of incident  $\gamma$ -quanta polarization,  $\vec{e}'$  is the vector of scattered  $\gamma$ -quanta polarization,  $*$  means complex conjugation,  $\vec{P}$  is the scatterer polarization vector,  $Q_{ik}$  is the rank two polarization tensor (quadrupolarization tensor) of scatterer,  $\vec{n}_{\gamma}$  is the unit vector along the  $\gamma$ -quantum momentum.

The term containing  $G$  ( $G\vec{P}$  is the gyration vector) describes effects of spin rotation and circular dichroism in the target with polarized nuclei (nucleons) [9], the term proportional  $\sim B$  describes  $\gamma$ -quanta birefringence in polarized target [9], the term containing  $\vec{n}_{\gamma}$  describes dependence of  $\gamma$ -quanta absorbtion on the orientation of quadrupolarization tensor  $Q_{ik}$  of scatterer (similarly the term  $(\vec{S}\vec{n})^2$  in (3)).

Now spin dichroism was experimentally observed for deuterons with the energy 10-20 MeV [10,11] passing through a carbon target. In these experiments the value  $Im d_1$  was found for the first time i.e. the difference in the deuteron scattering cross-sections  $\Delta\sigma = \sigma_{m=\pm 1} - \sigma_{m=0}$ , where  $\sigma_{m=\pm 1}$  is the total scattering cross-section for a deuteron with the spin projection onto  $\vec{n}$  is  $m = \pm 1$  and  $\sigma_{m=0}$  is the total scattering cross-section for a deuteron with the spin projection onto  $\vec{n}$  is  $m = 0$ . Tensor polarization of deuteron beam with the energy 5.5 GeV travelling through matter was observed in recent experiments [12], too.

It should be mentioned that  $\Delta\sigma \neq 0$  means that spin features of a vector meson produced inside a nucleus (another particle with the spin  $\geq 1$ ) will differ from spin properties of the particle produced by a stand-alone nucleon (which does not compose the nucleus) that results, in particular, in change of L/T-ratio.

To describe rescattering processes in the energy range, where  $Ref(0) \ll Imf(0)$ , the expressions obtained in [2] can be used. But the total cross-sections of vector meson production there should be replaced by  $\sigma_{M=\pm 1}$  or  $\sigma_{M=0}$ .

Additional analysis is necessary for  $Ref(0)$  comparable or larger than  $Imf(0)$ ; it will done separately.

It should be mentioned that in general case two correlations present in photoproduction: correlation  $[\vec{e}^* \vec{e}] \vec{J}$  (where  $\vec{J}$  is the operator of produced particle spin) is sensitive to circular polarization of photons and produced particle has vector polarization. Correlation  $(\vec{e} \vec{J})^2$ , which is sensitive to

linear polarization of photon, corresponds production of particle with tensor polarization. Due to birefringence effect produced particles are absorbed by the nucleus differently. Therefore, yield of vector-mesons depends on photon polarization i.e. production cross-section are different for different polarization of incident photons  $\sigma_{circ} \neq \sigma_{lin}$ .

In the present paper it is shown that spin-orbital interaction contributes to the birefringence effect for particles with the spin  $\geq 1$  along with central interaction.

Consideration is made by the example of contribution from spin-orbital interaction to birefringence effect, which is caused by interaction of particle with the coulomb field of the nucleus.

## 2 The amplitude of forward scattering for a particle with the spin $\geq 1$ in coulomb field

### 2.1 Spin dichroism caused by the spin-orbital electromagnetic interaction

Let us consider first scattering of a structureless charged particle with the spin  $\vec{S}$  by a coulomb center. The energy of spin-orbital interaction of such a particle the electric field can be expressed as follows:

$$\hat{V}_{em} = ib\vec{S}[\vec{E} \times \vec{\nabla}], \quad (5)$$

where  $b = (g - 2 + \frac{2}{\gamma+1})\mu_B \frac{\hbar}{mc\gamma^3}$ , nuclear Bohr magneton  $\mu_B = \frac{e\hbar}{2mc}$ ,  $g$  is the gyromagnetic ratio, for deuteron  $g = 1.72$ ,  $\gamma$  is the particle Lorentz factor. Therefore, the amplitude of particle scattering by electric field can be written in the first Born approximation as follows:

$$\begin{aligned} \hat{f}(\vec{k}' - \vec{k}) &= \hat{f}_c(\vec{k}' - \vec{k}) - \frac{m\gamma}{2\pi\hbar^2} \hat{V}_{em}(\vec{k}' - \vec{k}) = \hat{f}_c(\vec{k}' - \vec{k}) - \frac{m}{2\pi\hbar^2} b \vec{S}[\vec{E}(\vec{k}' - \vec{k}) \times \vec{k}] = \\ &= \hat{f}_c(\vec{k}' - \vec{k}) - \frac{m}{2\pi\hbar^2} b \Phi(\vec{k}' - \vec{k}) \vec{S}[\vec{k}' \times \vec{k}], \end{aligned} \quad (6)$$

where  $\hat{f}_c(\vec{k}' - \vec{k})$  is the amplitude of coulomb scattering of a charge by charge,  $\Phi(\vec{k}' - \vec{k})$  is the Fourier transform of the coulomb potential. For a shielded coulomb potential  $\Phi(\vec{k}' - \vec{k}) = \frac{4\pi Ze}{(\vec{k}' - \vec{k})^2 + \kappa^2} \rho(\vec{k}' - \vec{k})$ ,  $Ze$  is the scattering center charge,  $\kappa = \frac{1}{R_{sh}}$ ,  $R_{sh}$  is the shielding radius,  $\rho(\vec{k}' - \vec{k}) = \int e^{-i(\vec{k}' - \vec{k})\vec{r}} \rho(\vec{r}) d^3r$  is the Fourier transform of the charge distribution density in the scatterer  $\rho(\vec{r})$ . When charge distribution is spherically symmetric  $\rho(\vec{k}' - \vec{k}) = \rho(|\vec{k}' - \vec{k}|)$ .

From (6) follows that amplitude of forward scattering in the first Born approximation is determined only by Coulomb interaction of the charges of colliding particles, while contribution from spin-orbital interaction is zero.

Let us obtain now contribution from spin-orbital interaction to total scattering cross-section:

$$\sigma = \int f_{em}^+(\vec{k}' - \vec{k}) f_{em}^+(\vec{k}' - \vec{k}) d\Omega_{\vec{k}'} = \left(\frac{m\gamma}{2\pi\hbar^2}\right)^2 b^2 \int \Phi^2(\vec{k}' - \vec{k}) (\vec{S}[\vec{k}' \times \vec{k}])^2 d\Omega_{\vec{k}'}, \quad (7)$$

i.e.

$$\sigma = \left(\frac{m\gamma}{2\pi\hbar^2}\right)^2 b^2 \int \Phi^2(\vec{k}' - \vec{k}) (\vec{S}[\vec{k}' \times \vec{k}]) (\vec{S}[\vec{k}' \times \vec{k}]) d\Omega_{\vec{k}'} = \left(\frac{m\gamma}{2\pi\hbar^2}\right)^2 b^2 \int \Phi^2(\vec{k}' - \vec{k}) (\vec{k}'[\vec{k} \times \vec{S}]) (\vec{k}'[\vec{k} \times \vec{S}]) d\Omega_{\vec{k}'}, \quad (8)$$

Let us direct the axis  $z$  along the wavevector  $\vec{k}$ .

In this case  $\Phi^2(\vec{k}' - \vec{k}) = \Phi^2(\sqrt{k_{\perp}^2 + (k'_z - k^2)^2})$  i.e.  $\Phi^2$  does not depend on the azimuth angle  $\phi$  of vector  $\vec{k}$ ,  $k'_{\perp}$  is the component of  $\vec{k}'$  perpendicular to  $\vec{k}$ . Obviously  $\Phi^2(\vec{k}' - \vec{k}) = \Phi^2(\sqrt{2(1 - \cos\vartheta)})$ ,  $\vartheta$  is the azimuth angle of  $\vec{k}'$ . As a result, the expression in (8) can be integrated over  $\phi$  that gives:

$$\sigma = \left(\frac{m\gamma}{2\pi\hbar^2}\right)^2 b^2 k^4 \pi \int_0^\pi \Phi^2(\sqrt{2(1 - \cos\vartheta)}) \sin^2\vartheta (S(S+1) - S_z^2) \sin\vartheta d\vartheta. \quad (9)$$

From here it immediately follows that total cross-section  $\sigma_{M=\pm 1}$  for a particle with spin and magnetic quantum number  $M = \pm 1$  is not equal to total cross-section  $\sigma_{M=0}$  for a particle with spin having magnetic quantum number  $M = 0$ .

Difference  $\Delta\sigma$  can be expressed as:

$$\Delta\sigma = \sigma_{M=\pm 1} - \sigma_{M=0} = -\frac{m^2\gamma^2}{4\pi^2\hbar^4}b^2k^4\pi \int_0^\pi \Phi^2(\sqrt{2(1-\cos\vartheta)})\sin^3\vartheta d\vartheta. \quad (10)$$

Fourier transform of the Coulomb potential looks like:

$$\Phi(\vec{k}' - \vec{k}) = \frac{4\pi Ze}{(\vec{k}' - \vec{k})^2 + \kappa^2} = \frac{4\pi Ze}{2k^2(1 - \cos\vartheta) + \kappa^2} \quad (11)$$

Therefore, for particle scattered by the point coulomb center  $\rho(\vec{k}' - \vec{k}) = 1$

$$\Delta\sigma = -\frac{m^2\gamma^2}{4\pi^2\hbar^4}b^2k^4\pi \int_{-1}^1 \frac{(4\pi Ze)^2(1 - \kappa^2)}{[2k^2(1 - x) + \kappa^2]} dx. \quad (12)$$

Integration gives:

$$\Delta\sigma = \frac{m^2\gamma^2}{8\pi\hbar^4}b^2(4\pi Ze)^2 \left( 2 + \left( 1 + \frac{\kappa^2}{2k^2} \right) \ln \frac{\kappa^2}{2k^2(1 + \frac{\kappa^2}{2k^2})} \right) \quad (13)$$

It is interesting that for deuteron, having  $g = 1.72$  at the energy  $E \approx 11.5$  GeV from (13) it follows that  $\Delta\sigma = 0$ .

## 2.2 Particle spin rotation around the momentum due to spin-orbital electromagnetic interaction

According to [3] dispersion relations for the zero-angle scattering amplitude determines  $Re f_\pm(0) \neq Re f_0(0)$ .

As a result, spin-orbital interaction causes difference of refraction indices  $n_\pm - n_0 \neq 0$  and, therefore, birefringence effect (i.e. the effect of spin rotation around the direction of incident particle momentum) and spin dichroism.

To find the spin-dependent  $Re f(0)$  let us use the second Born approximation (note that according to the above in the first Born approximation spin-orbital interaction does not contribute to  $\hat{f}(0)$ ).

The scattering amplitude in the second Born approximation reads:

$$\hat{f}^{(2)}(0) = \frac{m^2\gamma^2}{8\pi^4\hbar^4}P \int \frac{\hat{V}^+(\vec{k} - \vec{q})V(\vec{q} - \vec{k})}{q^2 - k^2} d^3q + i\frac{m^2\gamma^2}{8\pi^3\hbar^4} \int \hat{V}^+(\vec{k} - \vec{q})V(\vec{q} - \vec{k})\delta(q^2 - k^2)d^3q, \quad (14)$$

where  $P$  indicates principal integral value. The imaginary part of amplitude  $Im \hat{f}^{(2)}(0) = \frac{k}{4\pi}\sigma$  and the expression for the cross-section  $\sigma = \frac{4\pi}{k}Im \hat{f}^{(2)}(0)$  certainly agrees with the expression (7).

Integration over the angles of  $\vec{q}$  provides for  $Re \hat{f}^{(2)}(0)$  the following expression:

$$\begin{aligned} Re \hat{f}^{(2)}(0) &= \frac{m^2\gamma^2}{8\pi^4\hbar^4}P \int \frac{b^2\Phi^2(\vec{q} - \vec{k})(\vec{S}[\vec{q} \times \vec{k}])^2}{q^2 - k^2} q^2 dq d\Omega_q = \\ &= \frac{m^2\gamma^2}{8\pi^3\hbar^4}b^2P \int \frac{q^4k^2}{q^2 - k^2} \Phi^2(q\sqrt{2(1 - \cos\vartheta)})(S(S+1) - S_z^2)\sin^3\vartheta d\vartheta dq, \end{aligned} \quad (15)$$

the axes  $z$  is directed along the momentum of the incident particle, i.e. along  $\vec{k}$ .

For particles with the spin 1 the real part of the difference in amplitudes  $f_{\pm 1}(0)$  and  $f_0(0)$  is expressed as:

$$Re\Delta f(0) = Ref_{\pm 1}^{(2)}(0) - Ref_0^{(2)}(0) = \frac{m^2 \gamma^2 b^2 k^2}{8\pi^3 \hbar^4} P \int \frac{q^4}{q^2 - k^2} \Phi^2(q\sqrt{2(1 - \cos\vartheta)}) \sin^3\vartheta d\vartheta dq, \quad (16)$$

Let us find expression for  $Re\Delta f$  for a particle scattered by a point Coulomb scatterer:

$$Re\Delta f(0) = \frac{m^2 \gamma^2 b^2 k^2}{8\pi^3 \hbar^4} P \int \frac{q^4}{q^2 - k^2} \frac{(4\pi Z e)^2}{[(q^2 - 2kq\cos\vartheta + k^2) + \kappa^2]^2} \sin^3\vartheta d\vartheta dq, \quad (17)$$

i.e.

$$Re\Delta f(0) = \frac{m^2 \gamma^2 b^2 k^2}{8\pi^3 \hbar^4} P \int_0^\infty \int_{-1}^{+1} \frac{q^4}{q^2 - k^2} (4\pi Z e)^2 \frac{1 - x^2}{[q^2 - 2kqx + k^2 + \kappa^2]^2} dx dq, \quad (18)$$

Integration over  $x$  can be done explicitly that provides:

$$Re\Delta f(0) = \frac{m^2 \gamma^2 b^2 k^2}{8\pi^3 \hbar^4} (4\pi Z e)^2 P \int_0^{q_{max}} \frac{q^4}{q^2 - k^2} J(q) dq, \quad (19)$$

where

$$J(q) = -\frac{1}{[(q^2 + k^2 + \kappa^2)^2 - 4k^2 q^2]} \left( \left( \frac{q^2 + k^2 + \kappa^2}{kq} - 2 \right) - \frac{1}{2k^2 q^2} - \frac{2(q^2 + k^2 + \kappa^2)}{4(kq)^3} \ln \frac{q^2 + k^2 + \kappa^2 - 2kq}{q^2 + k^2 + \kappa^2 + 2kq} \right), \quad (20)$$

The expression under the integration in (19) diverges on the upper limit as  $q^2$ . This growth is caused by the point nature of interaction in the considered example. If consider the function  $\rho(k - q)$ , this growth disappears.

### 3 Conclusion

Spin-orbital interaction contributes to the birefringence effect for particles with the spin  $S \geq 1$  along with central interaction. More detailed calculation of birefringence effect influence on the process of production (photoproduction) of particles with the spin  $S \geq 1$  in nuclei will be done separately.

### References

- [1] A. Dorokhov. "Production of Polarized Vector Mesons", In Abstracts of XII Workshop on High Energy Spin Physics, DSPIN2007 (Dubna, Russia, September 3-7, 2007) p.12.
- [2] Jörg Hüfner, Boris Kopeliovich, Jan Nemchik. *Phys. Lett.*, **B 383** (1996) 362.
- [3] V.G. Baryshevsky. *Phys. Lett.*, **A 171** (1992).
- [4] V.G. Baryshevsky. *J. Phys. G: Nucl. Part. Phys.*, **19** (1993) 273.
- [5] V.G. Baryshevsky, K.G. Batrakov and S.L. Cherkas. *J. Phys. G: Nucl. Part. Phys.*, **24** (1998) 2049.
- [6] Handbuch der Physic 39, 112 (Berlin: Springer 1957).
- [7] S.S. Gerstein and G.M. Zinoviev. *Sov. J. Nucl. Phys.* **33** (1981) 772.
- [8] M. L. Goldberger and K. M. Watson. "Collision Theory", Wiley (1964).
- [9] V.G. Baryshevsky. *Sov. J. Nucl. Phys.* **4** (1966) 1045.

- [10] V.G. Baryshevsky,C. Düweke,R. Emmerich et al., LANL e-print archive:hep-ex/0501045.
- [11] V.G. Baryshevsky,C. Düweke,R. Emmerich et al., Proceedings of the 17th International Spin Physics Symposium, SPIN2006, Kyoto, Japan, <http://www-nh.scphys.kyoto-u.ac.jp/SPIN2006/SciPro/pres/session6.7/AnatoliRouba.ppt>.
- [12] L.S. Azhgirey, V.P. Ladygin, A.V. Tarasov, L.S. Zolin. "Observation of tensor polarization of deuteron beam travelling through matter" In Abstracts of XII Workshop on High Energy Spin Physics, DSPIN2007 (Dubna, Russia, September 3-7, 2007) p.6.